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Mathematical model for flood routing in Jingjiang River and Dongting Lake network

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Abstract: The main stream of the Yangtze River, Dongting Lake, and the river network in the Jingjiang reach of the Yangtze River constitute a complex water system. This paper develops a one-dimensional (1-D) mathematical model for flood routing in the river network of the Jingjiang River and Dongting Lake using the explicit finite volume method. Based on observed data during the flood periods in 1996 and 1998, the model was calibrated and validated, and the results show that the model is effective and has high accuracy. In addition, the one-dimensional mathematical model for the river network and the horizontal two-dimensional (2-D) mathematical model for the Jingjiang flood diversion area were coupled to simulate the flood process in the Jingjiang River, Dongting Lake, and the Jingjiang flood diversion area. The calculated results of the coupled model are consistent with the practical processes. Meanwhile, the results show that the flood diversion has significant effects on the decrease of the peak water level at the Shashi and Chenjiawan hydrological stations near the flood diversion gates, and the effect is more obvious in the downstream than in the upstream.

Key words: *Jingjiang River; Dongting Lake; Jingjiang flood diversion area; flood routing; river networks; 1-D and 2-D coupled model*

1 Introduction

In the JingJiang reach of the Yangtze River, the main stream is linked to Dongting Lake at Songzi, Taiping and Ouchi river mouths, respectively, through Songzi, Hudu and Ouchi rivers. Besides, the inflows of the XiangJiang, Zishui, Yuanjiang, and Lishui rivers are drained into the Chenghan reach of the Yangtze River after being adjusted by Dongting Lake. The main stream, Dongting Lake, and some inflows are connected by a large number of river networks, forming a huge and complex system.

Due to its importance in flood control in the middle and lower Yangtze River, the Jingjiang reach has always been paid much attention in many studies. Zhong et al. (1996) established a flood routing model for the middle and lower Yangtze River. Tan et al. (1996)

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and Hu et al. (1996, 2002) developed a numerical model for the middle Yangtze River-Dongting Lake flood control system. Wu et al. (2003) constructed a mathematical model for simulating flow and sediment transport in the Jingjiang River-Dongting Lake network. Wu et al. (2004) proposed a one-dimensional implicit discrete numerical model for non-uniform sediment transport in river networks. Nakayama and Watanabe (2008) developed an integrated catchment-based eco-hydrology model to study the role of the flood storage capacity of lakes in the Yangtze River Basin. These research achievements have great significance to the flood control system of the middle Yangtze River and Dongting Lake.

As a technical support of the flood control in the middle Yangtze River and a reliable means of studying the relationship between the river and lake after the long-term operation of the Three Gorges Project, a one-dimensional (1-D) and two-dimensional (2-D) coupled model for flood routing in the JingJiang River and Dongting Lake is urgently needed. In this paper, a one-dimensional mathematical model for flood routing in the river network of the Jingjiang River and Dongting Lake was developed. This model was calibrated and validated using observed data during the flood periods in 1996 and 1998. Moreover, the one-dimensional mathematical model for the river network and the horizontal two-dimensional mathematical model for the Jingjiang flood diversion area were coupled.

2 1-D mathematical model for river network

2.1 Governing equations

The governing equations of the mathematical model of one-dimensional unsteady flow for river networks are composed of two parts (Li 1997; Zhu et al. 2001). One is the equations for the single channel between two nodes, and the other is the equations for nodes connecting two or more channels. The governing equations for a single channel are the continuity equation and momentum equation:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0 \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+gA\frac{\partial Z}{\partial x}+g\frac{n^2Q\left|Q\right|}{AR^{4/3}}=0 \tag{2}$$

where Q is the discharge, x is the distance along the mainstream, A is the area of wetted cross-section, t is time, g is the acceleration due to gravity, Z is the water level, n is the Manning's roughness coefficient, and R is the hydraulic radius.

The equations for nodes also include the continuity and momentum equations. The continuity equations at the mth node can be expressed as

$$Q_m+\sum_{l=1}^{L_m}Q_{m,l}=\frac{\partial \Omega_m}{\partial t} \qquad m=1,2,\cdots,K \tag{3}$$

where K is the total number of nodes in a river network, L_m is the amount of channels connected with the mth node, $Q_{m,l}$ is the discharge of the lth channel flowing into or out of the *m*th node, Q_m is the inflow discharge of the *m*th node excluding the discharge from the conflux channels, and Ω_m is the water storage of the *m*th node.

Generally, considering that the water level at the endpoint of every conflux channel around a node is the same, this condition is taken as a similarity of the momentum equations. Thus, the momentum equations at the nodes can be expressed as

$$Z_{m,1} = Z_{m,2} = \cdots = Z_{m,L_m} = Z_m \qquad m = 1, 2, \cdots, K \tag{4}$$

where $Z_{m,l}$ $(l=1, 2, \cdots, L_m)$ is the water level at the endpoint of the *l*th channel connected with the *m*th node, and Z_m is the water level of the *m*th node.

2.2 Discretization of governing equations

To discretize the above governing equations, the finite volume method was adopted, which has a characteristic of mass conservation. The first order upwind scheme was used to discretize the convective term in the momentum equation, because this numerical scheme has a high accuracy in solving nonlinear convection and convection-diffusion equations (Zhang et al. 2001; Zhang et al. 2003). The discrete equations of a single channel are shown as follows:

$$A_{i+1/2}^{k+3} = A_{i+1/2}^{k+1} - \frac{4\Delta t}{\Delta x_i + \Delta x_{i+1}}\left(Q_{i+1}^{k+2} - Q_i^{k+2}\right) \tag{5}$$

$$\frac{Q_i^{k+2} - Q_i^k}{2\Delta t} + g\frac{A_{i+1/2}^{k+1} + A_{i-1/2}^{k+1}}{2}\frac{Z_{i+1/2}^{k+1} + Z_{i-1/2}^{k+1}}{\Delta x_i} + g\frac{n_i^2\dfrac{Q_i^{k+2} + Q_i^k}{2}\left|Q_i^k\right|}{\dfrac{A_{i+1/2}^{k+1} + A_{i-1/2}^{k+1}}{2}\left(\dfrac{R_{i+1/2}^{k+1} + R_{i-1/2}^{k+1}}{2}\right)^{4/3}} =$$

$$-\begin{cases} \dfrac{u_i^k Q_i^k - u_{i-1}^k Q_{i-1}^k}{\left(\Delta x_{i-1} + \Delta x_i\right)/2} & u_i^k \geq 0,\ u_{i-1}^k \geq 0 \\ \dfrac{u_i^k Q_i^k}{\left(\Delta x_{i-1} + \Delta x_i\right)/2} & u_i^k > 0,\ u_{i-1}^k \leq 0 \\ \dfrac{u_{i+1}^k Q_{i+1}^k - u_i^k Q_i^k}{\left(\Delta x_i + \Delta x_{i+1}\right)/2} & u_i^k \leq 0,\ u_{i+1}^k < 0 \\ \dfrac{-u_i^k Q_i^k}{\left(\Delta x_i + \Delta x_{i+1}\right)/2} & u_i^k < 0,\ u_{i+1}^k \geq 0 \end{cases} \tag{6}$$

where Δx is the space step, Δt is the time step, the subscript *i* denotes the cross-section serial number, and the superscript *k* denotes the serial number of the time step, *u* is the velocity of a cross-section.

The explicit scheme was applied in the above equations. Therefore, the water level of each single channel and node could be solved directly, avoiding the computation difficulties in solving the large coefficient matrixes generated in the previous river network models.

2.3 Calibration and validation of 1-D mathematical model for river network

In the Jingjiang reach of the Yangtze River, the main stream, Dongting Lake and branch channels constitute a complex water system (Zhang et al. 2010). In this study, the water system was generalized as a one-dimensional river network model, which includes 52 nodes, 72 channels and 1 176 cross-sections, as shown in Fig. 1.

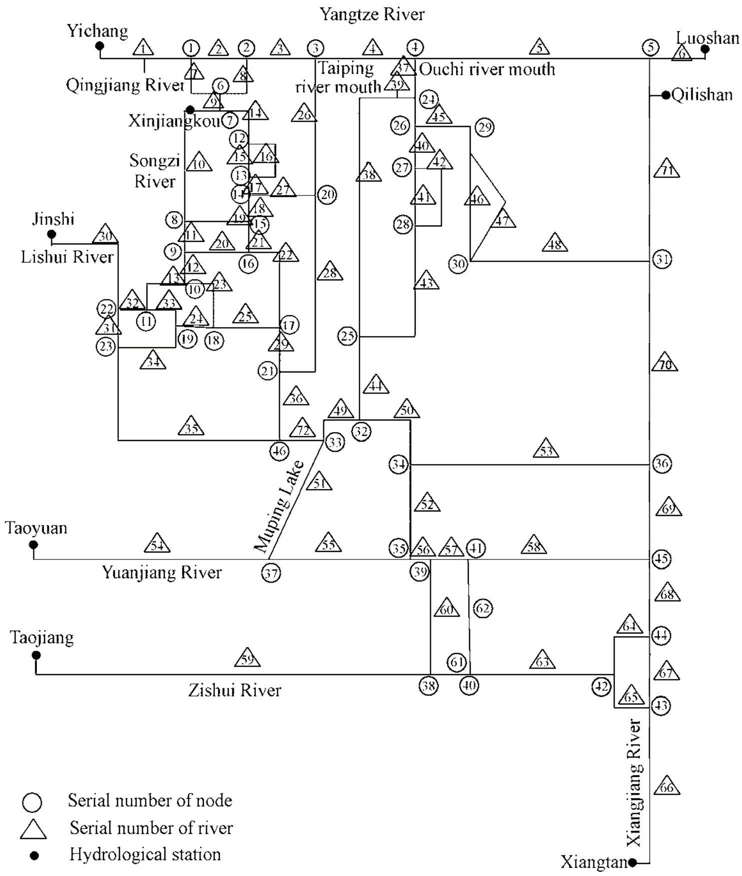


Fig. 1 Sketch of river network structure

In order to check the integrity and accuracy of the model algorithm together with the rationality of some treatment techniques, hydrological data from June 1 to August 30, 1996 in combination with the field topography data of the main stream in October, 1998 and Dongting Lake in October, 1995 were used to calibrate the one-dimensional river network model. The discharge processes of the Yichang, Xiangtan, Taojiang, and Taoyuan hydrological stations were given as the upper boundary conditions, and the water level process of the Luoshan Hydrological Station was given as the downstream boundary condition.

The calibrated Manning's roughness coefficients range from 0.016 to 0.03 in the main stream, 0.02 and 0.035 in Dongting Lake as well as the three distributary channels: the Songzi, Taiping, and Ouchi channels. The Manning's roughness coefficients were graded based on grading the water level in the model calculation to avoid obvious differences in different flood processes.

Then, the calibrated Manning's roughness coefficients were used in the following model validation, where the flood process in the Yangtze River during June to September of 1998 was simulated. The comparison between the computational and measured results at the Xinjiangkou, Luoshan, and Qilishan hydrological stations is shown in Fig. 2 and Fig. 3. As can be seen from these figures, the calculated results including the flood rising and falling process and the flood peak value agree well with the measured data, which shows that the model has high accuracy and can be used for flood routing in the Jingjiang River and Dongting Lake areas.

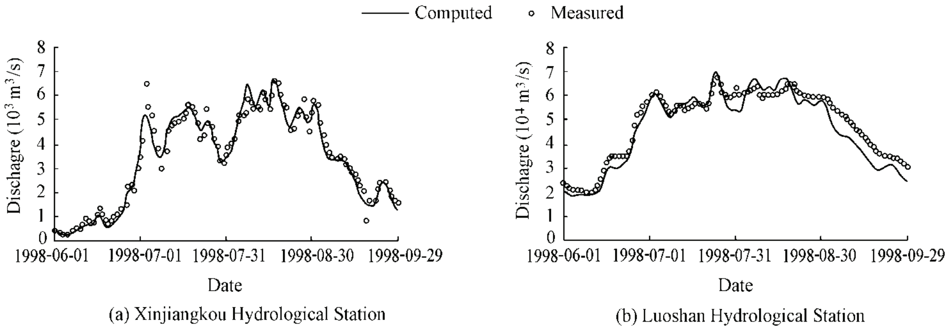


Fig. 2 Discharge process during flood season (June to September) in 1998

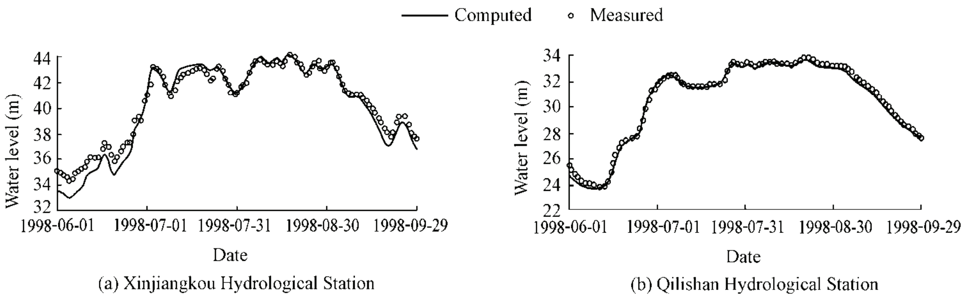


Fig. 3 Water level process during flood season (June to September) in 1998

3 Application of 1-D and 2-D coupled model

The Jingjiang flood diversion project, located on the southern bank of the Jingjiang River, is a main part of the Jingjiang River flood control system, with an area of 921 km^2 and an effective storage capacity of 5.4 km^3. When the one-dimensional hydrodynamic model is used for flood routing in the Jingjiang River, it can not simulate the changes of the mainstream water level and discharge if the Jingjiang flood diversion project is put into use. Meanwhile, the duration of flood, submerged depth and flow field can be got only if the two-dimensional flood routing model is applied. Based on the calculated results of the two-dimensional model, corresponding flood management plans can be made.

In order to simulate the flood process in the Jingjiang River, Dongting Lake, and Jingjiang flood diversion area, a one-dimensional mathematical model and a two-dimensional mathematical model need to be coupled. The one-dimensional model was applied to the river network of the Jingjiang River and Dongting Lake, and the two-dimensional mathematical model was applied to the Jingjiang flood diversion area.

3.1 Horizontal 2-D mathematical model for flood routing

The governing equations of the horizontal two-dimensional mathematical model for flood routing are composed of flow continuity and momentum equations (Wang and He 1999; Hong et al. 2001; Hua and Xu 2003; Xu and Yin 2003; Yang et al. 2007; Yu and Zhang 2001):

$$\frac{\partial z}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0 \tag{7}$$

$$\frac{\partial M}{\partial t}+\frac{\partial (UM)}{\partial x}+\frac{\partial (VM)}{\partial y}=-gh\frac{\partial (h+z_{\mathrm{b}})}{\partial x}-\frac{gn^2U\sqrt{U^2+V^2}}{h^{1/3}}+\gamma_{\mathrm{t}}\left(\frac{\partial^2 M}{\partial x^2}+\frac{\partial^2 M}{\partial y^2}\right) \tag{8}$$

$$\frac{\partial N}{\partial t}+\frac{\partial (UN)}{\partial x}+\frac{\partial (VN)}{\partial y}=-gh\frac{\partial (h+z_{\mathrm{b}})}{\partial y}-\frac{gn^2V\sqrt{U^2+V^2}}{h^{1/3}}+\gamma_{\mathrm{t}}\left(\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}\right) \tag{9}$$

where h is the water depth, U and V are the velocities in x and y directions, respectively, $M=Uh$, $N=Vh$, z is the water level, z_{b} is the riverbed elevation, and γ_{t} is the turbulent viscosity coefficient.

In order to satisfy the law of mass conservation in flow, the finite volume method and conservation scheme were adopted to discrete the governing equations. Besides, the staggered grids and the leap frog method were used in which the water depth and velocity components were computed at different nodes and time levels. The detailed discretization process of the model can be found in Zhang and Yu (2001) and Xie et al. (2006). The horizontal two-dimensional mathematical model has been used for dike burst simulation, dam-break flood simulation of the hydropower station, and flow movement simulation in the flood diversion area. A new irregular boundary treatment method, namely the diagonal Cartesian method which was proposed by Mu and Zhang (2006) and was improved by Mu et al. (2006), was used to fit the complex boundary in the flood diversion area in this study.

3.2 Mode of water quantity exchange between channel and flood diversion area

The flood diversion area is connected with the channel by the flood diversion gate, and the discharge through the flood diversion gate is usually described using the broad crest weir formula. Because the weir formula and the continuity equation for a channel have the same form of computational expression after they are discretized, the flood diversion area can be regarded as the water storage node, which is linked with other channels through the flood diversion gates. The discharge through the flood diversion gate is calculated in the river network, and then the water level inside the embankment can be got after the flood is regulated by the reservoir. Obviously, the reservoir-type flood regulation method neglects flood routing in the flood diversion area, therefore, the flood regulation process is to some extent different from the practical physical process. Particularly for the flood diversion area with complicated topography, the simulation accuracy of this method is not high.

In this study, according to the characteristics of the horizontal two-dimensional mathematical model, the water level at the flood diversion gate was used as the external boundary condition of the two-dimensional model, which was provided by the results of one-dimensional river network model, and then the discharge through the flood diversion gate could be computed automatically. Based on this, the flood process in the flood diversion area was simulated. The water quantity exchange between the flood diversion area and the channel was conducted by the explicit junction method. The method can be described by the following equations:

$$Z^{*}B = \int_{0}^{B} z^{*}\mathrm{d}y \tag{10}$$

$$Q^{*} = \int_{0}^{B} u^{*}h^{*}\mathrm{d}y \tag{11}$$

where Z^{*} and Q^{*} are the water level and discharge of the one-dimensional cross section at the interface between the channel and flood diversion area, B is the width of the interface, z^{*} is the water level of grids at the interface, and u^{*} and h^{*} are the mean velocity and water depth of grids at the interface, respectively.

The above-mentioned method can be applied to the flood diversion area with several flood diversion gates and unclosed computation boundary. It can also avoid the disadvantage of reservoir-type flood regulation method, and the computation is more rational because the information is exchanged at the interface between the flood diversion area and the channel. Moreover, it can simulate the flood process and submerged conditions well. Hence, it can provide a basis for assessment of economic loss in the flood diversion area.

3.3 Calculated results of coupled model

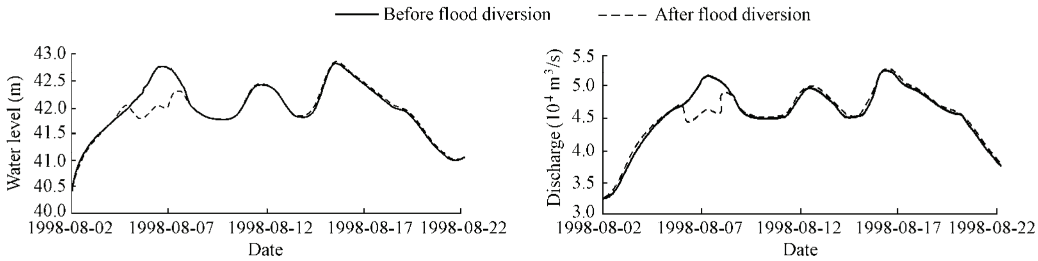
Based on a measured topographic map of the Jingjiang flood diversion area in May, 1995, the computational domain was divided into 225 × 450 grids. Each grid has a length of 160 m and a width of 180 m.

On August 7, 1998, the water level at the Shashi Hydrological Station exceeded 44.67 m, which was above the guaranteed water level. To check the effect of the Jingjiang flood diversion area on the decrease of the water level and discharge at the Shashi Hydrological Station, the one- and two-dimensional coupled mathematical model was applied, giving that the flood diversion gates were opened on August 5 and 6, 1998, to let the flood in.

The comparison of peak water level before and after the flood diversion is shown in Table 1. The comparisons of water level and discharge processes at the Shashi Hydrological Station are shown in Fig. 4. The Chenjiawan and Shashi hydrological stations are about 10 km in the upstream and downstream of the flood diversion gate, respectively, and the peak water levels of these two stations decreased obviously by 0.79 m and 0.78 m, respectively. Besides, the peak water level decreased by 0.67 m at the Shishou Hydrological Station, which is located in the downstream of the Shashi Hydrological Station, and 0.48 m at the Jianli Station, which is located in the most downstream. However, for the Yichang and Zhicheng hydrological stations, which are located in the upstream of the Chenjiawan Hydrological Station, the peak water level decreased unconspicuously. Overall, the flood diversion has significant effects on the decrease of the peak water levels at the Shashi and Chenjiawan hydrological stations which are near the flood diversion gates, and the effect is more obvious in the downstream than in the upstream. The analysis indicates that the calculated results of the model are in reasonable accordance with the physical processes.

Table 1 Comparison of peak water levels at hydrological stations before and after flood diversion m

Hydrological station (from upstream to downstream)	Peak water level		Decrease of peak water level
	Before flood diversion	After flood diversion	
Yichang	51.65	51.60	0.05
Zhicheng	47.88	47.78	0.10
Chenjiawan	43.29	42.50	0.79
Shashi	42.76	41.98	0.78
Shishou	38.82	38.15	0.67
Jianli	36.27	35.79	0.48



(a) Water level process (b) Discharge process

Fig. 4 Water level and discharge processes at Shashi Hydrological Station

Fig. 5 shows the flow field at 200 h after the flood diversion gates were opened. It can be seen that the coupled model can simulate the flood process in the Jingjiang flood diversion area well.

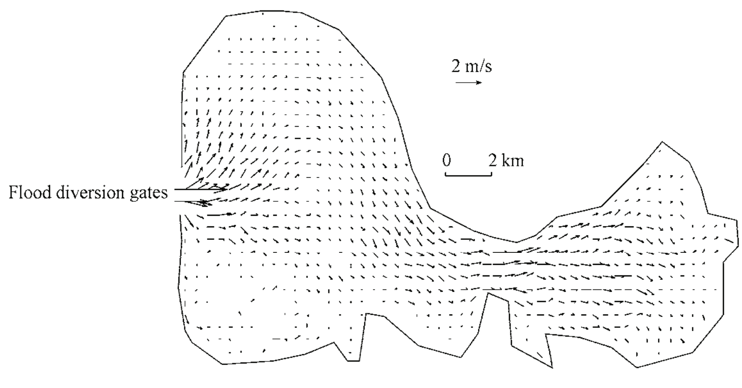


Fig. 5 Flow field in Jingjiang flood diversion area at 200 h after opening flood diversion gates

4 Conclusions

(1) A one-dimensional mathematical model for flood routing in the Jingjiang River and Dongting Lake network was established using the explicit finite volume method. Based on observed data during the flood periods in 1996 and 1998, the model were calibrated and validated, and the results show that the model is effective and has high accuracy.

(2) The one-dimensional mathematical model was coupled with the horizontal two-dimensional mathematical model to simulate the flood process in the Jingjiang River, Dongting Lake, and the Jingjiang flood diversion area. The simulated results of the coupled model are in reasonable accordance with the practical physical processes.

(3) The flood diversion has significant effects on the decrease of the peak water level at the Shashi and Chenjiawan hydrological stations near the flood diversion gates, and the effect is more obvious in the downstream than in the upstream.

(4) The coupled model will play an important role in the flood control in the middle Yangtze River. Considering the dramatic changes of the riverbed terrain in the Jingjiang River and Dongting Lake, together with the greatly reduced incoming sediment after the operation of the Three Georges Project, and automatical adjustment of hydraulic elements according to the erosion and deposition on the riverbed, the established model in this study should be further improved to simulate sediment transport and riverbed deformation, thus this model can be supposed to be more effective and reliable.

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